view factor from the disk to differential area dA_1 is given by

$$dF_{A_2-dA_1}=\frac{dA_1}{\pi R^2}F_{dA_1-A_2}$$

Then, the view factor from the disk to a finite surface can be calculated from

$$F_{A_2-A_1}=\int_{A_1}dF_{A_2-dA_1}=\int_{A_1}F_{dA_1-A_2}\frac{dA_1}{\pi R^2}$$

Conclusion

In conclusion, a general formulation for diffuse configuration factors from planar point sources to disks is presented here. This formulation can be used to evaluate view factors for all possible positions of differential areas relative to disks. The present formulation can be reduced to the existing formulation by assigning numerical values to a number of its parameters. Finally, the resulting equations can be used to evaluate view factors from disks to finite surfaces.

References

[1]Sparrow, E.M., "A New and Simpler Formulation for Radiative Angle Factors," *Transactions of ASME, Journal of Heat Transfer,* Vol. 85, No. 2, 1963, pp. 81–88.

[2]Chung, B.T.F. and Sumitra, P.S., "Radiation Shape Factors from Plane Point Sources," *Transactions of ASME, Journal of Heat Transfer,* Vol. 94, No. 3, 1972, pp. 328–330.

[3]Juul, N.H., "Diffuse Radiative View Factors from Differential Plane Sources To Spheres," *Transactions of ASME, Journal of Heat Transfer,* Vol. 101, No. 3, 1979, pp. 558–560.

[4]Naraghi, M.H.N. and Chung, B.T.F., "Radiation Configuration Factors between Disks and a Class of Axisymmetric Bodies," *Transactions of ASME, Journal of Heat Transfer,* Vol. 104, No. 3, 1982, pp. 426–431.

[5]Howell, J.R., *A Catalog of Radiation Configuration Factors,* McGraw-Hill, New York, 1982.

[6]Siegel, R. and Howell, J.R., *Thermal Radiation Heat Transfer,* 2nd ed., McGraw-Hill, New York, 1981.

Cavitation of Liquid Streams in a Vacuum

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Introduction

As our activities in space increase, there is a growing requirement for accurate predictions of the results of injecting finite vapor pressure liquids into the space environment. Liquids may be exposed to space because of either accidents or of planned dumps. During experimental studies at the DFVLR Institute for Rarefied Gases at Göttingen,[1] the following behavior of a water jet in a vacuum was observed. The water jet that was formed in a tapered nozzle leaves it as a collimated jet and abruptly bursts into ice particles and water droplets downstream, forming a conical volume filled with stream fragments. Both the distance at which the bursting takes place and the angle defined by the cone of stream fragments depend on flow parameters such as the initial water temperature, the nozzle diameter, and the stagnation pressure. In this paper, comparisons are made between the authors' interpretations of the data obtained from the DFVLR and predictions of the fragment cone's angle and the bursting point's location for water jets injected into a vacuum.

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Analysis

Phenomena that occur when a volatile liquid stream is injected into a vacuum have been studied by Fuchs and Legge[1] and Muntz and Orme.[2] For the purposes of this paper, we assume an axisymmetric cylindrical water jet emanating from a contoured nozzle as shown. The stream has a speed V_0, which is associated with the stagnation pressure p_0. As the fluid enters the vacuum, the pressure of the surface of the jet drops to about half of the vapor pressure of the fluid associated with the temperature of the jet. An increase in the evaporation rate that is a result of the rapid pressure drop causes the jet to cool at the surface while moving downstream from the nozzle exit. Despite surface cooling (which leads to a lowered vapor pressure) and the retarding effect of internal pressure generated by surface tension, a vapor bubble may expand in the stream under certain conditions of initial bubble size, stream diameter, and initial stream temperature. As a bubble continues to grow, it eventually reaches a critical size that causes the jet to burst into water droplets and ice fragments. The fragments form a conical envelope where the apex is defined as the burst point.

The equation of motion for the radius R_B of a bubble in a viscous liquid as a function of time is (see Ref. 3):

$$R_B\ddot{R}_B+\frac{3}{2}\dot{R}_B^2=\frac{\Delta p}{\rho_\ell}-\frac{2\sigma}{\rho_\ell R_B}-\frac{4\mu\dot{R}_B}{\rho_\ell R_B} \tag{1}$$

where

$$\Delta p=p_{vB}-\frac{\sigma}{a_0}-\frac{p_v}{2} \tag{2}$$

is the pressure difference driving the bubble expansion. This particular form of Δp is for a cylindrical stream and bubbles with vapor pressure p_{vB}. Here, σ is the surface tension, μ the coefficient of viscosity, ρ_ℓ the density of the liquid, and a_0 the stream radius. The vapor pressure p_v is a function of the temperature and is given by the Clausius–Clapeyron equation.

The reduction of surface temperature with time due to vaporization of both the external stream and the interior bubble wall were calculated by using an approximation that we developed for the integral relation for surface temperature due to Shultz and Jones as given by Fuchs and Legge.[1] The approximation is decribed in Ref. 2.

Equation (1) can be solved straightforwardly with the boundary condition that $\hat{R}_B=0$ at $\hat{t}=0$. Typical results of $\hat{R}_B$ plotted against $\hat{t}$ are illustrated in Ref. 2. Here, $\hat{R}_B$ is the nondimensional bubble radius (R_B/a_0) and $\hat{t}$ is the nondimensional time equal to tV_0/a_0, where V_0 is the stream speed and a_0 the stream radius.

Results

Bursting Cone Angles

To calculate the characteristic "cone angles" produced when a stream bursts, we have assumed that the bursting takes place when a bubble grows to be the size of the stream. We have assumed that the perpendicular particle velocity is equal to the slope of the bubble growth curve, $\hat{R}_B$ versus $\hat{t}$, when the bubble radius is the same as the stream radius. Hence, if the stream speed is V_0 and the perpendicular fragment velocity is $\dot{R}_B$, then the maximum cone angle that can be observed from a side-view of the stream is just $\theta = 2\tan^{-1}(\hat{\dot{R}}_B)$ at $\hat{R}_B = 1$, where $\hat{\dot{R}}_B$ is the nondimensional bubble growth rate $\dot{R}_B/V_0$. This expression for the cone angle reduces to the expression derived by Wu, Steinberger, and Bracco[4] if we assume that in their case only one bubble is present and we consider conditions at the edge of the stream.

We have examined the effect of particle acceleration due to the expanding vapor in a preliminary way. The vapor density and velocity distributions of the vapor field away from the cylindrical jet were assumed to take the form as given by Knight,[5] and by numerically integrating the force exerted on the particle, the change in speed of the particle can be determined. The approximate calculations for a single particle injected into the vapor field show that the perpendicular particle velocity may increase by over a factor of two, affecting substantially the burst angle. The calculations are meant to give an estimate of the significance of the acceleration of the particle by expanding vapor field, and it is understood that the actual structure may be different from the model used. A detailed analysis of this phenomenon requires the knowledge of burst particle size, which most likely varies with temperature, the fragment spatial distribution, and the fragment burst temperatures. The complexity of these issues necessitates extensive further research.

Experiments at the DFVLR were made by injecting a water stream into a vacuum chamber, observing and photographing the behavior of the jet as it either traveled unaffected in the vacuum environment or as it burst into ice fragments and water droplets. Nozzle geometry and diameter, stream velocity, initial water temperature, and gas content were variable parameters in the studies. In this paper, we consider for comparison to prediction only those data acquired with the conically tapered nozzle (which has a converging taper half-angle of 8.1 deg and a length to diameter ratio of 9.9 to 11.6 depending on nozzle diameter), demineralized water, and without the inclusion of air bubbles. A comparison of the DFVLR data and the aforementioned calculations are shown in Fig. 1. We have also obtained data made available by Bednarz[6] who measured burst angle and burst distance from video-recorded data of release events of water dumping on STS mission 61-B. The video tape was obtained from NASA's Photography and Television Division. The data point from this source is indicated with a cross. The curves on the plot are the predicted natural logarithm of the burst angle vs initial stream temperature for 1.5 mm diam streams traveling at 11.8 m/s. The solid curve represents the predictions without the effect of the expanding vapor i.e., $\ln\theta = \ln[2\tan^{-1}(\hat{\dot{R}}_B)]$ at $\hat{R}_B = 1$. The broken line represents the prediction of the cone angle with the effect of particle acceleration included and with the vapor temperature the same as the temperature of the stream when it burst, i.e., the cooled surface temperature. The dotted line is the prediction that also includes the effect of particle acceleration by the expanding vapor with the vapor temperature the same as the initial stream temperature, i.e., the uncooled internal stream temperature. The two cases considering particle acceleration are meant to bracket the region of actual burst angle, as the particle temperature will most likely be somewhere between the cooled surface temperatures and the uncooled internal temperature. For both cases considering particle acceleration, the particle size was assumed to be one-tenth the size of the stream. The key result of the comparison is that the cone angles decrease with decreasing temperature, as can be seen in both experiment and prediction, and that the effect of particle acceleration is significant. The discrepancy in the slope of the experiment and prediction may possibly be explained by particle size. In the analysis given, the particle size was assumed constant with temperature. It seems reasonable to suggest that the particles should decrease with increasing temperature. If this trend were taken into account, the slope of the prediction would shift toward the slope of the data. It is clear that more experiments need to be done in order to estimate the particle size with temperature, and the burst particle's temperature with flow properties.

An experiment similar to the one carried out at the DFVLR was done by Steddum, Maples, and Donovan[7] in which they illuminated outgassed water streams at bursting and from photographs calculated observed cone angles. Our calculations are in rough agreement with their experiments, except that they found a significant stream diameter dependence, whereas our calculations without effects of vapor acceleration are insensitive to stream diameter. However, if particle acceleration due to the expanding vapor field is included in our calculations, we find a significant stream diameter dependence. If the particle size is held constant, the predictions show that decreasing the stream diameter decreases the burst angle. Steddum et al. found that decreasing the stream diameter increased the burst angle. It should be mentioned here however, as was also mentioned by Steddum et al., that it is difficult to interpret cone angles from photographs. This is because the burst fragments are dense near the centerline of

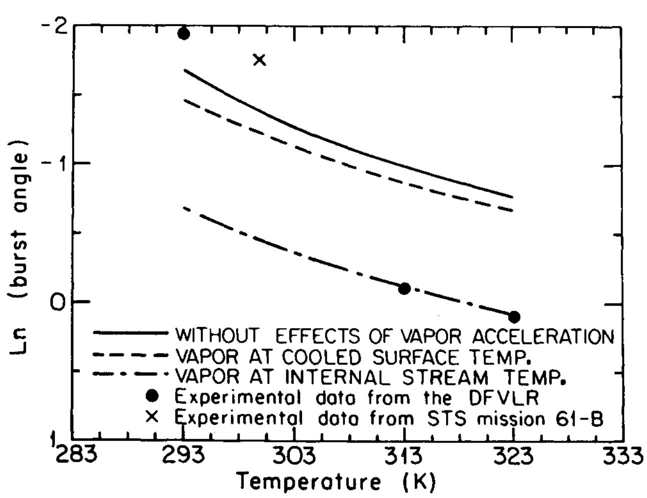

Fig. 1 A comparison of calculated to observed burst angle.

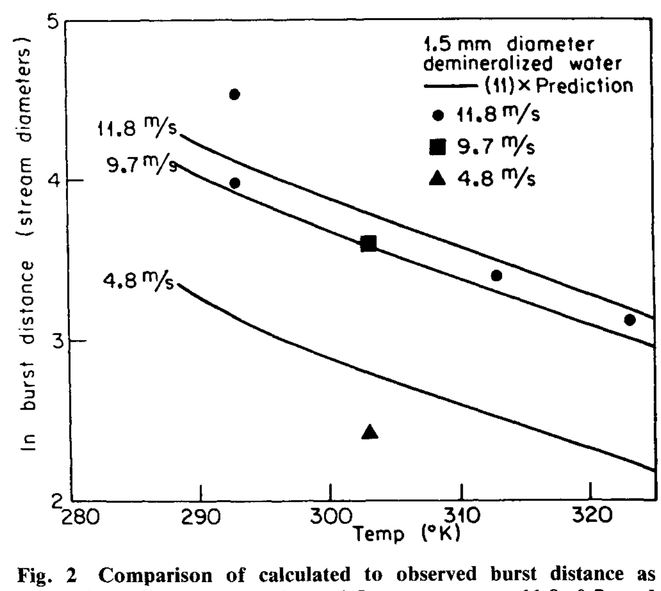

Fig. 2 Comparison of calculated to observed burst distance as a function of temperature for a 1.5 mm stream at 11.8, 9.7, and 4.8 m/s.

the jet, and they become progressively less dense away from the axis of the jet. As a result, it is frequently difficult to obtain a clear definition of the boundary of the fragment cone. It is also not known what particle sizes were measured in the experiments of Ref. 7.

Burst Distances

We have taken measurements from the DFVLR data described in the previous section to estimate the burst distance of several streams at different temperatures. Some of these results have been plotted in the form of ℓn (burst distance) as a function of temperature as shown in Fig. 2. Our predictions agree with the slope of the data, but the calculated distances are shorter by a constant factor. We have plotted the natural logarithm of the predicted burst distance and have included the constant factor that best fits the data. The constant factor included in the predictions may possibly be explained by hypothesizing some effective nucleation distance that is not accounted for in our analysis (and seems to be much longer than the growth distance) in which the bubbles are developing, and they begin to grow sometime later downstream. With the inclusion of the constant factor, there is generally good agreement of prediction with experiment. It can be seen that although there is significant scatter in the data available, the burst distance decreases with increasing temperature and with decreasing velocity.

Figure 3 further illustrates the dependence of the burst distance on the stream's speed for a fixed initial temperature. The closed circles are true data points, the open circle is an interpolated point obtained from Fig. 2, the cross is the data point obtained from video-recorded data from the STS mission 61-B, and the curve is the prediction including the constant multiplier. Again, the slope of the curve is nearly identical to prediction, and it is clear that increasing the stream speed moves the burst point further downstream.

Summary

The limiting trajectory of macroscopic particles from cavitating water streams has been predicted. In experiments where we were able to examine the original data, the predictions were quite successful. From these studies, it was found that a water stream may exist in bundled form in a vacuum without bursting if the initial water temperature is sufficiently low. Also, it was determined that increasing the stream speed increases the burst distance. Fragment cone angles of the bursts generally increase with increasing water temperature, as seen in both experiment and prediction. It was also found that acceleration of the particles due to the expanding vapor may affect significantly the burst angle. Experiments need to be done to measure particle size distribution as a function of position in the plumes of cavitating streams in a vacuum, and as a function of stream temperature in order to accurately predict the acceleration of the burst particles, thus having the ability to accurately predict the cone angle. Although some recent results of this nature[8,9] are now available for water, they are for streams that have cavitated close to the nozzle throat. The cavitation locations of Refs. 8 and 9 are in substantial disagreement (greater than one order of magnitude) with the burst distances observed in the experiments reported in Ref. 1.

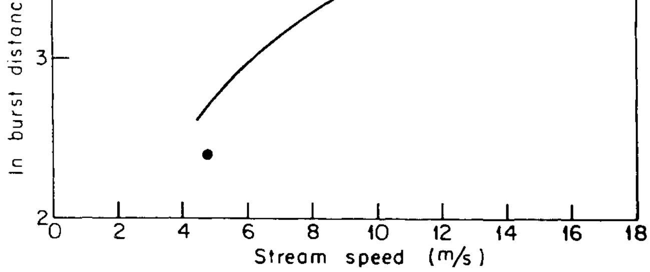

Fig. 3 Comparison of calculated to observed burst distance as a function of stream velocity for a 1.5 mm stream and an initial temperature of 303K. • Experimental data from DFVLR, o interpolated data from DFVLR, X Experimental data from STS mission 61-B.

Acknowledgment

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References

[1]Fuchs, H. and Legge, H., "Flow of Water Jet into Vacuum," *Acta Astronautica,* Vol. 6, 1979, pp. 1213–1226.

[2]Muntz, E.P. and Orme, M., "The Characteristics, Control, and Uses of Liquid Streams in Space," *AIAA Journal* (submitted for publication).

[3]Van Stralen, S. and Cole, R., *Boiling Phenomena,* edited by S. Van Stralen and R. Cole, McGraw-Hill, New York, 1979, Vol. 1, Chap. 7, p. 197.

[4]Wu, K.J., Steinberger, R.L., and Bracco, F.V., "On the Mechanism of Breakup of Highly Superheated Liquid Jets," The Combustion Institute Central States Section, Spring Meeting, Warren, MI, 1981.

[5]Knight, C.J., "Evaporation from a Cylindrical Surface into Vacuum," *Journal of Fluid Mechanics,* Vol. 75, 1976, pp. 469–486.

[6]Bednarz, J., "Astro Innovations," (private communication) Feb. 1987.

[7]Steddum, F., Maples, D., and Donovan, M., "Investigation on the Characteristics of Fluids Vented into Vacuum," NBS Space Simulation 56, 905–914, 1970.

[8]Curry B.P., Bryson, R.J., Seiber, B.L., Jones, J.H., Kiech, E.L., and Powell, H.M., "Preliminary Results of the AEDC Liquid Venting Experiment," *Proceedings of the 14th JANNAF Plume Technology Meeting,* CPI Publication 384, Vol. 2, Nov. 1983, pp. 105–116.

[9]Curry, B.P., Bryson, R.J., Seiber, B.L., and Kiech, E.L., "Additional Results from an Experiment Venting and H_2O Jet into a High Vacuum," Calspan Corporation, AEDC Division, Final Rept. AD B096121, June 1985.

Perturbation Analysis of Tapered Fins with Nonlinear Thermal Properties

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Introduction

THE use of perturbation techniques for the analysis of heat conduction in fins with nonlinear thermal properties may be found in several recent studies. Perturbation methods are computationally efficient and are sufficiently accurate for fin design purposes. Indeed, Aziz and Na[1] have devoted an entire book to the subject of perturbation methods for heat transfer problems.

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